Excitation of nonlinear two-dimensional wake waves in radially nonuniform plasma

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It is shown that an undesirable curvature of the wave front of a two-dimensional nonlinear wake wave, excited in uniform plasma by a relativistic charged bunch or laser pulse, may be compensated by radial change of the equilibrium plasma density. [S1063-651X(99)06510-1]

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The progress in the technology of ultrahigh intensity lasers and high-current relativistic charged bunch sources permits the use of laser pulses [1] or charged bunches [2] for excitation of strong plasma waves. The excited plasma waves can be used for both acceleration of charged particles and focusing of bunches to get high luminosity in the linear colliders [2]. At present, the plasma-based accelerator concepts are being actively developed, both theoretically and experimentally (see the overview in Ref. [3] and references therein).

The amplitude of longitudinal electric field $E_{\max}$ in relativistic wake waves excited in cold plasma is limited by the relativistic wave-breaking field [4] $E_{rel}=[2(\gamma-1)]^{1/2}E_{WB}/\beta$, where $\gamma=(1-\beta^2)^{-1/2}$ is a relativistic factor, $\beta=v_{ph}/c$ is a dimensionless phase velocity of the wave, $E_{WB}=m_e\omega_{pe}v_{ph}/e$ $(E_{WB}[V/\mathrm{cm}]\approx 0.96n_p^{1/2}[\mathrm{cm}^{-3}])$ is the conventional nonrelativistic wave-breaking field, $\omega_{pe}=(4\pi n_p e^2/m_e)^{1/2}$ is the electron plasma frequency, n_p is the equilibrium density of plasma electrons, m_e and e are the mass and absolute value of the electron charge. The acceleration rate in the wake fields can reach tens of GeV/m, which much exceeds the rates reached in conventional accelerators.

The linear wake-field theory is valid when $E_{\max}\ll E_{WB}$. In the case of wake wave excitation by relativistic charged particle bunch [plasma wake-field accelerator (PWFA)] this corresponds to the condition $\alpha=|q|n_b/en_p\ll 1$[2], where q is the charge of bunch particles, n_b is their concentration. In the scheme of wake wave excitation by a short laser pulse [of the length comparable with plasma wavelength $\lambda_p=2\pi v_{ph}/\omega_{pe}$; laser wake-field accelerator (LWFA)], the linear theory is valid when $a^2=e^2E_0^2/(m_e^2c^2\omega_0^2)\ll 1$ [5], where $\omega_0\gg\omega_{pe}$ and E_0 are the frequency and amplitude of laser radiation, respectively. The phase velocity of the wake wave is equal to the bunch velocity v_b in PWFA and to the laser pulse group velocity $v_g\approx c$ $(1-\omega_{pe}^2/2\omega_0^2)$(that corresponds to $\gamma=\gamma_g\approx\omega_0/\omega_{pe}\gg 1$) in LWFA.

The one-dimensional nonlinear wake waves excited by wide drivers (when $k_pr_d\gg 1$, where $k_p=\omega_{pe}/v_{ph}$ is the wave number and r_d is the radius of the driver) are studied in sufficient detail both for PWFA [6] and LWFA [7]. These studies testify to the feasibility of excitation of strong nonlinear plasma waves with an amplitude of up to E_{rel} by bunches with $\alpha\gtrsim 0.5$ and laser pulses with $a^2\gtrsim 1$. The other important results of the one-dimensional nonlinear theory are the steepening of the wake wave and the increase of wavelength with amplitude. The wave with the amplitude $E_{\max}\approx E_{rel}$ has a wavelength nearly $\gamma^{1/2}$ times as large as the linear wavelength λ_p.

In reality, the transverse sizes of the drivers used are ordinarily comparable or less than their longitudinal size. The allowance for finite transverse sizes of the drivers and, accordingly, the transverse motion of plasma electrons, complicate the treatment of the problem in the nonlinear regime. In the general case, the analytical solution of this regime seems impossible, and here the use of numerical methods are usually required. The numerical investigation of nonlinear effects in two-dimensional (axially symmetrical) wake waves excited in uniform plasma has shown that in the nonlinear regime the wavelength becomes dependent on the transverse (relative to the driver propagation direction) coordinate [3,8–10]. The change of a nonlinear wavelength in the transverse direction is due to the dependence of the wavelength on the amplitude, that, in its turn, is varied in the radial direction owing to the finite cross section of the driver. This leads to a curvature of the phase front of the nonlinear wave [8–10], to steepening and ''oscillations'' of the field in the transverse direction [10,11] and eventually to the development of turbulence. From the viewpoint of acceleration and focusing of charged bunches in the wave, the curvature of the nonlinear wave front is undesired as the quality (emittance, monochromaticity) of the driven bunch worsens. In the present work we show that by means of wake wave excitation in plasma, the density of which is properly varied in the transverse direction, one can eliminate the nonlinear change of wavelength in the transverse direction and the related curvature of phase front. The plasma of this kind may be produced by charged beams [12] or laser pulses [13] passing through a neutral gas or a partially ionized uniform plasma due to an additional ionization.

Equations for nonzero components of plasma electrons momentum and electromagnetic field describing the steady nonlinear wake fields in radially nonuniform plasma can be obtained by a simple generalization of equations for uniform plasma [10,11,14,15]:

$$\beta\frac{\partial P_z}{\partial z}-\frac{\partial\gamma_e}{\partial z}-\beta^2E_z=0, \tag{1}$$

$$\beta\frac{\partial P_r}{\partial z}-\frac{\partial\gamma_e}{\partial r}-\beta^2E_r=0, \tag{2}$$

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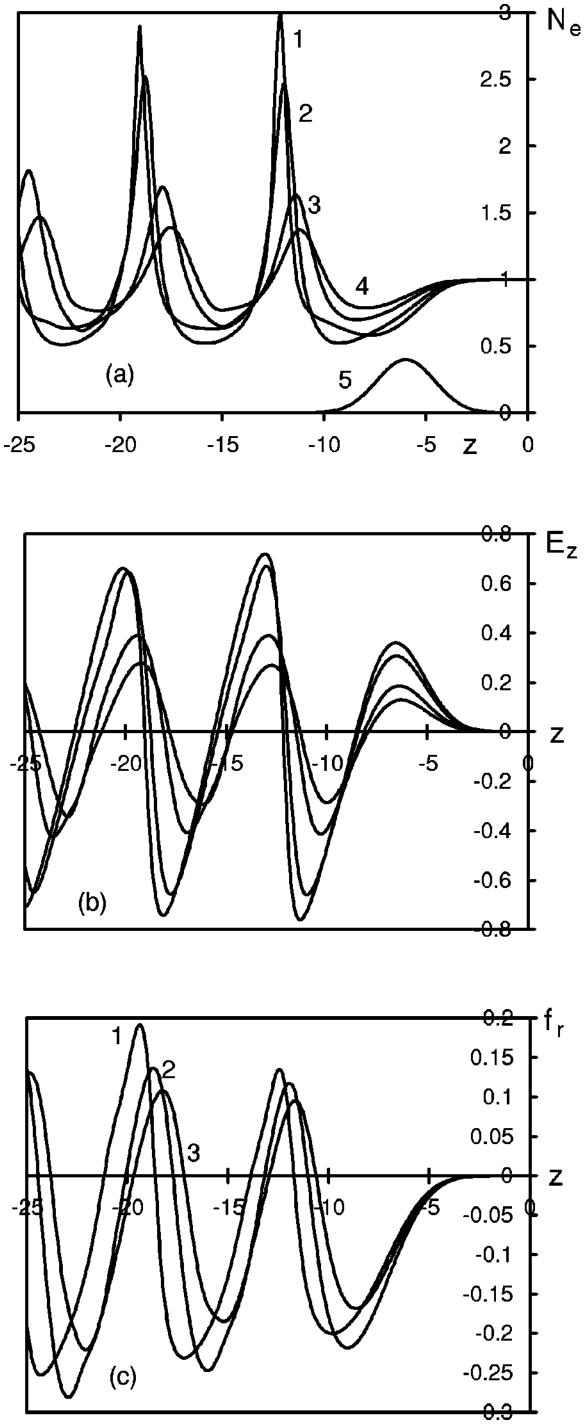

FIG. 1. The two-dimensional nonlinear wake wave in uniform plasma $[N_p(r)=1\ ]$. The parameters of the bunch are $\alpha_0=0.4$, $\sigma_z=2$, $\sigma_r=5$, $\gamma=10$. (a) The density of plasma electrons N_e and of the bunch. Curve 1: the density of plasma electrons at the axis, $r=0$; Curve 2: the same for $r=2$; Curve 3: $r=4$; Curve 4: $r=5$; Curve 5: the density of bunch at the axis $\alpha(z,r=0)$. (b) The longitudinal electric field for $r=0, 2, 4$, and 5 in the order of magnitude reduction. (c) The focusing field $f_r=\beta H_\theta-E_r$. Curve 1: $r=2$; Curve 2: $r=4$; Curve 3: $r=5$. All variables are normalized.

$$-\frac{\partial H_\theta}{\partial z}+\beta\frac{\partial E_r}{\partial z}+\beta_r N_e=0, \tag{3}$$

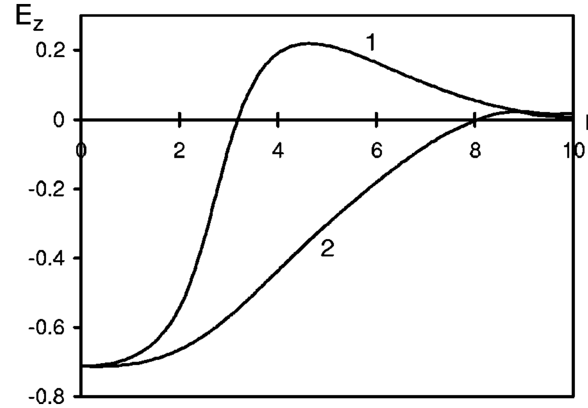

FIG. 2. The radial behavior of the normalized longitudinal electric field strength E_z. Curve 1: $E_z(z=-25,r)$ in the nonlinear wake wave excited in uniform plasma for the case given in Fig. 1 $(|\Delta z|\approx\xi)$ [see Eq. (9)]; Curve 2: $E_z(z=-25,r)$ in nonuniform plasma for the case given in Fig. 3.

$$\nabla_\perp H_\theta+\beta\frac{\partial E_z}{\partial z}+\beta_z N_e+\beta\alpha=0, \tag{4}$$

$$\beta\frac{\partial H_\theta}{\partial z}-\frac{\partial E_r}{\partial z}+\frac{\partial E_z}{\partial r}=0, \tag{5}$$

$$N_e=N_p(r)-\alpha-\nabla_\perp E_r-\frac{\partial E_z}{\partial z}. \tag{6}$$

As usual, Eqs. (1) and (2) were derived taking into account the conservation of generalized momentum $\beta^2\mathbf{H}-\mathrm{rot}\mathbf{P=0}$, or in our case

$$\beta^2 H_\theta+\frac{\partial P_z}{\partial r}-\frac{\partial P_r}{\partial z}=0. \tag{7}$$

In Eqs. (1)–(6), $\gamma_e=(1+P_z^2+P_r^2+a^2/2)^{1/2}$, $\beta_{z,r}=P_{z,r}/\gamma_e$, and $N_e=n_e/n_p(r=0)$ are, respectively, a relativistic factor, dimensionless components of velocity, and dimensionless density of plasma electrons, $N_p=n_p(r)/n_p(0)$, $\beta=v_{ph}/c$, $z=k_p(r=0)(Z-v_{ph}t)$, $\nabla_\perp=\partial/\partial r+1/r$. Also, the following dimensionless variables have been used: the space variables are normalized on $\lambda_p(r=0)/2\pi=1/k_p(r=0)$, the momenta and velocities, respectively, on $m_e c$ and the velocity of light and the strengths of electric and magnetic fields, on the nonrelativistic wave-breaking field at the axis $E_{WB}(r=0)=m_e\omega_{pe}(r=0)v_{ph}/e$. The field of forces acting on relativistic electrons in the excited field is $\mathbf{F}(-eE_z,-e(E_r-\beta H_\theta),0)$. In PWFA $\alpha\neq0$, $a^2=0$ and in LWFA $\alpha=0$, $a^2\neq0$ [in Eqs. (1)–(6) the linear polarization of the laser pulse field is assumed; for the circular polarization the value of a should be multiplied by the factor $2^{1/2}$].

We have solved Eqs. (1)–(6) numerically, choosing the Gaussian profile of the driver both in longitudinal and transverse directions:

$$A(z,r)=A_0\exp[-(z-z_0)^2/\sigma_z^2]\exp(-r^2/\sigma_r^2), \tag{8}$$

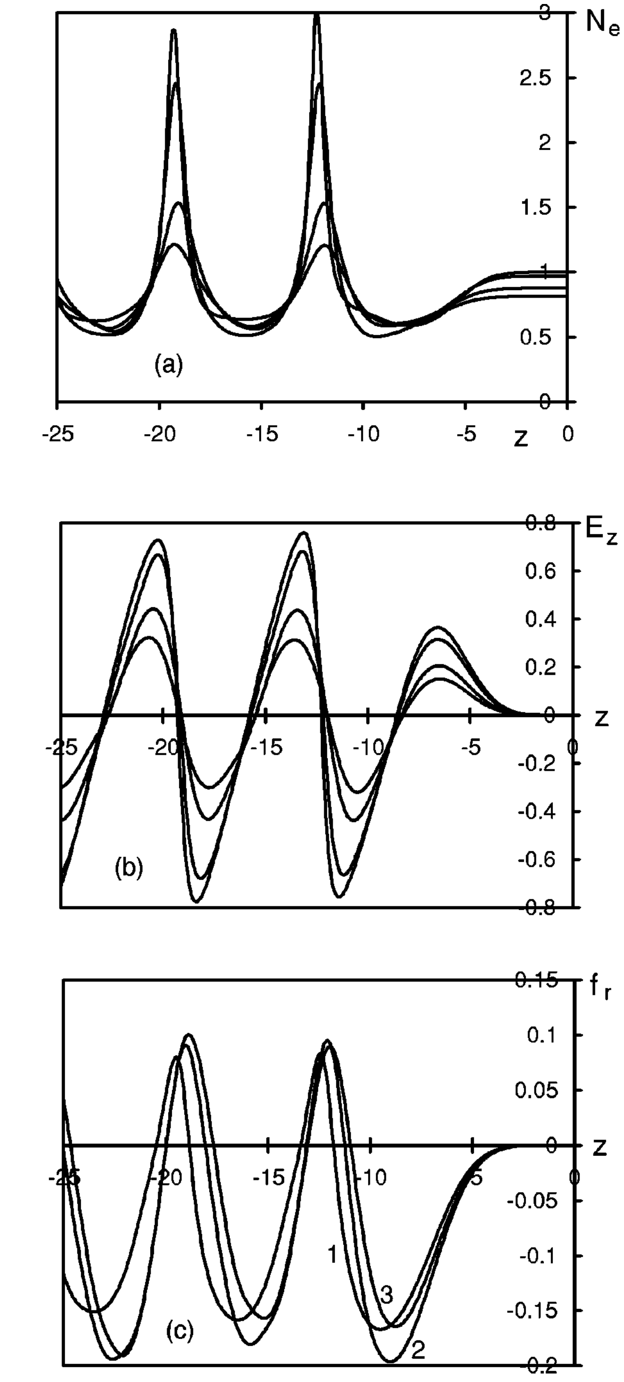

FIG. 3. The 2D nonlinear wake wave in nonuniform plasma with $\sigma_p=11$. The bunch parameters are the same as in Fig. 1. (a) The density of plasma electrons versus z for $r=0$, 2, 4, and 5 in the order of magnitude reduction. (b) The same for the longitudinal electric field. (c) The focusing field. Curve 1: $r=2$; Curve 2: $r=4$; Curve 3: $r=5$. All variables are normalized.

where $A(z,r)$ stands for $\alpha=n_b(z,r)/n_p(r=0)$ or $a^2(z,r)$. Shown in Fig. 1 is the nonlinear two-dimensional (2D) plasma wake wave excited in uniform plasma $[N_p(r)=1]$ by the relativistic electron bunch ($\alpha_0=0.4$, $\sigma_z=2$, $\sigma_r=5$; for example, in this case $n_{b0}=4\times10^{13}$ cm^{-3} and the characteristic longitudinal and transverse sizes of the bunch $\sigma_{z,r}/k_p$ correspondingly are 1.06 and 2.65 mm, when $n_p=10^{14}$ cm^{-3}). One can see that the wavelength changes with the radial coordinate r. This leads to curving of the phase front and to ''oscillations'' in the transverse direction (see Fig. 2, curve 1). As $|z|$ increases, the change of phase in transverse direction (for fixed z) becomes more and more marked. The longitudinal space parameter characterizing the nonlinear wave front curving is [10]:

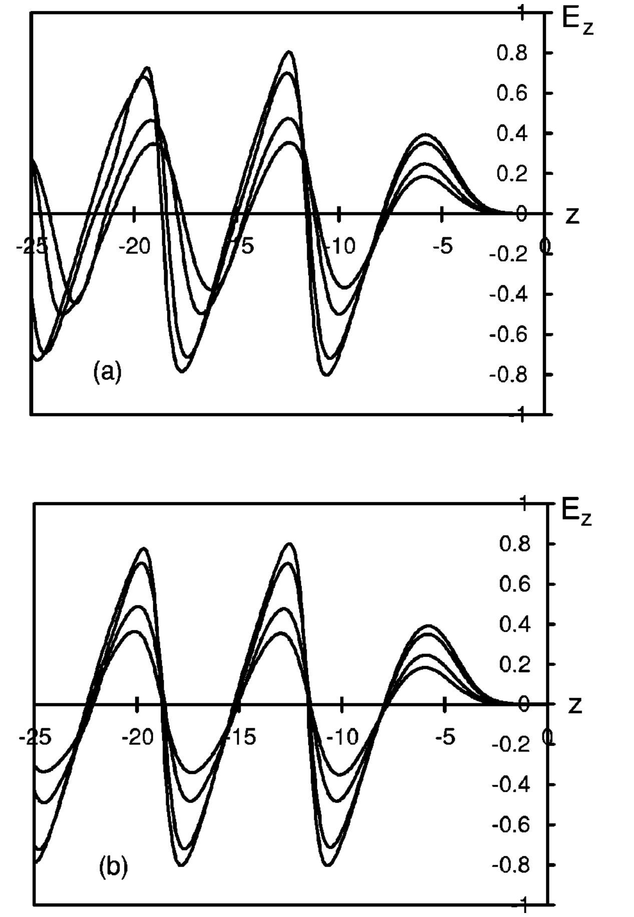

FIG. 4. The two-dimensional nonlinear wake wave excited by laser pulse. The pulse parameters are $a_0^2=3.6$, $\sigma_z=2$, $\sigma_r=5$, $\gamma=10$. (a) The dimensionless accelerating field E_z excited in uniform plasma. $r=0$, 2, 4, and 5 in the order of magnitude reduction. (b) The same in the nonuniform plasma, $\sigma_p=12$.

$$\xi=\frac{\lambda_p}{2[1-\lambda_p/\Lambda(0)]}, \tag{9}$$

where $\Lambda(r)$ is the nonlinear wavelength. At the distance $|\Delta z|\approx\xi$ from the driver, the oscillation phase at the axis ($r=0$) is opposite to that on the periphery ($r\gtrsim\sigma_r$). Thus, in 2D nonlinear regime the nonlinear wavelength changes with r due to the nonlinear increase of the wavelength with wave amplitude. On the other hand, the linear wavelength $\lambda_p\sim n_p^{-1/2}$ decreases with equilibrium density of plasma. Hence follows an opportunity to compensate the nonlinear increase in wavelength by reducing the wavelength that is due to the growth of equilibrium density of plasma. Indeed, assume that the nonlinear wavelength of the 2D wake wave in the uniform plasma $\Lambda(r)$ is known. Then, one can roughly compensate for the radial variation of the nonlinear wavelength by changing the equilibrium density of plasma in the radial direction according to the relation

$$\Lambda(0)/\Lambda(r)=\lambda_p(r)/\lambda_p(0)=[n_p(0)/n_p(r)]^{1/2}. \quad (10)$$

In this case, the equation for equiphase surfaces is $z\approx$const, and, therefore, the solution for fields could be written in the form $f_1(z)f_2(r)$. If we assume that the function $\Lambda(r)$ is Gaussian [that is approximately the case at least for $r<\sigma_r$, according to numerical data for profiles (8)], then one can take the transverse profile of the equilibrium plasma density to be also Gaussian:

$$n_p(r)=n_{p0}\exp(-r^2/\sigma_p^2). \quad (11)$$

From Eqs. (10) and (11) it follows that

$$\sigma_p=r/[\ln(\Lambda(0)/\Lambda(r))]^{1/2}. \quad (12)$$

For example, according to Eq. (12), the numerical data for $\Lambda(r)$ in the nonlinear wave shown in Fig. 1 give $\sigma_p\approx 11$. Thus, in the radially nonuniform plasma, the density of which is changed according to Eqs. (11) and (12), one can practically avoid undesirable curvature of the wave front of a nonlinear wave. Figures 3 and 4 illustrate the validity of this assertion, respectively, for PWFA and LWFA (see also Fig. 2, curve 2). One can see that the nonlinear wavelength in the nonuniform plasma with proper radial profile, does not practically change in the transverse direction.

As for the case of LWFA, one has to note that, as is well known, without optical guiding, the diffraction limits the distance of laser-plasma interaction (and, hence, the energy gain of particles accelerated by a wake wave) to a few Rayleigh lengths $Z_R=\pi r_0^2/\lambda$, where r_0 is the minimum laser pulse spot size at the focal point and λ is the laser wavelength. For high-intensity laser pulses, the quantity Z_R is usually of the order of several millimeters. One of the approaches in preventing diffraction broadening of the laser pulse and the increase of laser-plasma interaction distance, is the guiding of the pulse in a preformed plasma density channel [3]. Here, the unperturbed plasma density grows from the pulse axis $(r=0)$ to its periphery, in contrast with the radial profile of the plasma density that was proposed above, for prevention of phase front curvature of a nonlinear wake wave when the plasma density at the driver axis is at a maximum. In the case of nonlinear wake wave excitation by high power laser pulses (of power $P>P_c=2c(e/r_e)^2[\omega_0/\omega_{pe}(r=0)]^2\approx 17[\omega_0/\omega_{pe}(r=0)]^2 GW$, where $r_e=e^2/m_ec^2$ is the classical electron radius), the process of diffraction broadening of a larger part of pulse, including the case of proposed equilibrium profile of plasma, may be prevented or essentially retarded due to the relativistic self-focusing[3]. One can estimate the condition of relativistic self-focusing in our case from the following relation (see, e.g., Sec. VI in Ref. [3]): $P/P_c>1+(\Delta n/\Delta n_c)(r_s/r_0)^4$, where r_s is the spot size, $\Delta n_c=1/\pi r_e r_0^2$, $\Delta n\simeq n_p(0)-n_p(\sigma_r)$ is the radial variation of unperturbed plasma density in the laser pulse.

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